

# *P*-Wave Contact Tensor – Universal Properties of Axisymmetry-Broken *P*-Wave Fermi Gases

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We investigate universal properties of a *p*-wave Fermi gas with a resonant interaction in which the axisymmetry is broken spontaneously or externally. Here, the short-range correlations can be completely characterized by the nine-component *p*-wave contact tensor, which can be measured by applying a generalized adiabatic sweep theorem. The distinctive features of the *p*-wave contact tensor emerge in a normal *p*-wave Fermi gas in an anisotropic trap and in a superfluid phase. An experimental scheme to measure the *p*-wave contact tensor and test the adiabatic sweep theorem is also discussed.

Universal properties in ultracold atomic gases with resonant interactions have been gathering great interest. Among such universal properties are universal relations in the BCS-BEC crossover [1–5], which predict universal power laws in the short-range correlations such as a high-momentum asymptote of the momentum distribution and relate them to macroscopic properties such as a thermodynamic function. They hold in the resonant regime at any temperature, in the normal or superfluid phase, and in a trapped or uniform system. Here, a single quantity called Tan’s contact characterizes both short- and long-distance properties of the system. These theoretical predictions have also been verified experimentally [6–8].

Recently, such universal relations have been investigated in a Fermi gas with a *p*-wave Feshbach resonance; the notion of the contact has been extended [9–11] and experimentally measured [12]. A new feature that the *p*-wave interaction introduces is anisotropy; a magnetic field that controls the Feshbach resonance lifts the three-fold degeneracy of the *p*-wave resonance into two distinct ones via the magnetic dipole-dipole interaction [13]. This can make system’s correlation functions anisotropic, and the three-component *p*-wave contact has been introduced in Refs. [10, 11], which is justified when the system possesses the axisymmetry. However, breaking of the axial symmetry often plays an crucial role in this system. For example, the axial symmetry can be spontaneously broken in a *p* + *iβp* superfluid phase [14, 15]. Another example is a *p*-wave gas confined in an anisotropic trap, in which external fields may break the symmetry. Such a geometry has been utilized to investigate phenomena such as a confinement-induced resonance [16, 17].

The purpose of this Letter is to point out that, once we allow the axisymmetry breaking, a complete characterization of the short-range correlations requires the nine-component *p*-wave contact tensor  $C_{m,m'}$  [18]. For exam-

ple, the momentum distribution behaves as

$$n_{\mathbf{k}} \sim k^{-2} \sum_{m,m'=-1}^1 C_{m,m'} Y_1^{m*}(\hat{\mathbf{k}}) Y_1^{m'}(\hat{\mathbf{k}}) \quad (1)$$

for  $|\mathbf{k}| \gg k_F, \lambda_T$ , where  $k_F$  is the Fermi momentum, and  $\lambda_T$  is the thermal de Broglie length. This implies that  $C_{m,m'}$  contains the three-component *p*-wave contact as the three diagonal components. We show the following generalized adiabatic sweep theorem:

$$C_{m,m'} = \frac{32\pi^2 M}{\hbar^2} \frac{\partial E}{\partial (-1/v_{m,m'})}, \quad (2)$$

where  $v_{m,m'}$  defined later determines the low-energy scattering phase shift of transitions between the channels with the projections of the relative angular momentum  $m$  and  $m'$ . The diagonal component  $v_{m,m}$  is the usual *m*-dependent *p*-wave scattering volume which can be controlled by a *p*-wave Feshbach resonance. On the other hand,  $v_{m,m'}$  with  $m \neq m'$  is associated with an unconventional *p*-wave scattering that does not conserve the projection of the relative angular momentum of two colliding atoms, the consequences of which have not been investigated yet. There are no such processes in ordinary *p*-wave Fermi gases because the *p*-wave Feshbach resonance is anisotropic but still axially symmetric. However, we show that a Raman process in the  $\Lambda$  scheme (see Fig. 1) can be used to control the unconventional *p*-wave scattering, test the adiabatic sweep theorem and measure the entire *p*-wave contact tensor. We also demonstrate that the off-diagonal components of  $C_{m,m'}$  emerge in the *p* + *iβp* superfluid phase and a normal Fermi gas in a pancake-shaped trap.

First, let us discuss why the *p*-wave contact has to be promoted to a tensor. To be specific, we base our discussion on the following two-channel model of a single-component Fermi gas with a resonant *p*-wave interac-

tion [9]:

$$\begin{aligned} \hat{H} = & \int d^3\mathbf{r} \frac{1}{2} |\nabla \hat{\psi}(\mathbf{r})|^2 \\ & + \int d^3\mathbf{r} \left[ \frac{1}{4} |\nabla \hat{\phi}_m(\mathbf{r})|^2 + \epsilon_{m,m'} \hat{\phi}_m^\dagger \hat{\phi}_{m'}(\mathbf{r}) \right] \\ & + \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \left[ \frac{g}{2} u_m(\mathbf{r}_{12}) \hat{\psi}^\dagger(\mathbf{r}_1) \hat{\psi}^\dagger(\mathbf{r}_2) \hat{\phi}_m(\mathbf{R}_{12}) \right. \\ & \left. + \frac{g^*}{2} u_m^*(\mathbf{r}_{12}) \hat{\phi}_m^\dagger(\mathbf{R}_{12}) \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r}) \right], \end{aligned} \quad (3)$$

where summation over  $m, m' = -1, 0, 1$  is implied. Here  $\hat{\psi}$  is a fermionic field operator representing the open-channel atoms,  $\hat{\phi}_m$  is a bosonic operator representing the closed-channel molecules,  $\mathbf{r}_{12} \equiv \mathbf{r}_1 - \mathbf{r}_2$ , and  $\mathbf{R}_{12} \equiv \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}$ . We set  $\hbar = k_B = M = 1$ , where  $\hbar$  is the Planck constant divided by  $2\pi$ ,  $k_B$  is the Boltzmann constant, and  $M$  is the mass of the atom. The closed-channel energy tensor  $\epsilon_{m,m'}$  and a coupling constant  $g$  control the Feshbach resonance, and are determined so that they reproduce the  $p$ -wave scattering volume and the effective range. In the presence of a magnetic field parallel to the  $z$  axis,  $\epsilon_{m,m'}$  is diagonal and the Hamiltonian reduces to the familiar expression of the two-channel model [14]. In particular, the anisotropy of the  $p$ -wave Feshbach resonance due to the magnetic field can be taken into account in the  $m$ -dependence of  $\epsilon_{m,m}$ . We use the tensor here to make the expression of the Hamiltonian unchanged under coordinate rotation. The coupling function  $u_m(\mathbf{r})$  regularizes the short-range singularity of the interaction and is normalized so that its Fourier transform  $\tilde{u}_m(\mathbf{k})$  is  $\sim k Y_1^m(\hat{\mathbf{k}})$  for  $k \ll r_0^{-1}$ , where  $r_0$  is the range of the interaction.

Following Ref. [9], we introduce a set of wave functions  $\{\Psi^{(N_o, N_c)}\}$  characterizing a many-body state. Each  $\Psi^{(N_o, N_c)}(\{\mathbf{p}\}; \{\mathbf{q}\}, \{m\})$  represents a state with  $N_o$  open-channel atoms and  $N_c$  closed-channel molecules, in which  $\{\mathbf{p}\}$ ,  $\{\mathbf{q}\}$ , and  $\{m\}$  are the sets of the atomic momentum  $\mathbf{p}_i$ , the molecular momentum  $\mathbf{q}_i$ , and the angular momentum projection  $m_i$  of the molecular rotation, respectively. If we expand the time-independent Schrödinger equation in terms of  $1/p_i$ , we obtain

$$\begin{aligned} \Psi^{(N_o, N_c)}(\{\mathbf{p}\}; \{\mathbf{q}\}, \{m\}) \sim & \sum_{j=1, j \neq i}^{N_o} \sum_{m_{ji}=-1}^1 (-1)^{j+1} g \\ & \frac{Y_1^{m_{ij}}(\hat{\mathbf{p}}_{ij})}{p_{ij}} \Psi^{(N_o-2, N_c+1)}(\{\mathbf{p}\}_{ij}; \{\mathbf{q}\}_{ij}, \{m\}_{ij}) \end{aligned} \quad (4)$$

to the leading order. Here,  $\mathbf{p}_{ij} \equiv \frac{\mathbf{p}_i - \mathbf{p}_j}{2}$ ,  $\{\mathbf{p}\}_{ij}$  is  $\{\mathbf{p}\}$  excluding  $\mathbf{p}_i$  and  $\mathbf{p}_j$ ,  $\{\mathbf{q}\}_{ij}$  is  $\{\mathbf{q}\}$  including  $\mathbf{p}_i + \mathbf{p}_j$ , and  $\{m\}_{ij}$  is  $\{m\}$  including  $m_{ij}$ . This result indicates that an asymptotic wave function is in general a linear combination of terms proportional to the spherical harmonic function  $Y_1^m(\hat{\mathbf{p}}_{ij})$ . When calculating the momentum distribution, two things should be noted. First, the momentum distribution is calculated from the squared wave

function, which contains terms like  $Y_1^{m*}(\hat{\mathbf{p}}_{ij}) Y_1^{m'}(\hat{\mathbf{p}}_{ij})$ . Second, for terms with  $m \neq m'$  to vanish, the axisymmetry of the microscopic interaction is not sufficient, but the state has to be invariant under rotation around the  $z$  axis. Since we do not make the assumption of the axial invariance here, the momentum distribution has in general the high-momentum tail of the form in Eq. (1) up to the leading order.

We also find from Eq. (4) the following expression of  $C_{m,m'}$  within our two-channel model:

$$C_{m,m'} = |g|^2 \int \frac{d\mathbf{k}}{(2\pi)^3} \langle \hat{\phi}_m^\dagger(\mathbf{k}) \hat{\phi}_{m'}(\mathbf{k}) \rangle. \quad (5)$$

This expression gives useful interpretation of  $C_{m,m'}$  in the ultracold atomic experiments as the number of the closed-channel molecules in an  $m$  state for  $m = m'$  and the coherence between two molecular states  $m$  and  $m'$  for  $m \neq m'$ . The  $p$ -wave contact tensor belongs to the  $3 \otimes 3$  representation of the rotation group. Therefore, it can be decomposed into one-, three-, and five-dimensional irreducible representations, each of which can also be interpreted as the number, angular momentum, and nematicity of the closed-channel molecules, respectively.

The  $p$ -wave contact tensor is directly related to the thermodynamics through the adiabatic sweep theorem. To derive the theorem, we need to define the generalized  $p$ -wave scattering volume  $v_{m,m'}$ . If the  $p$ -wave scattering from an  $m$  state into an  $m'$  state is allowed, the scattering amplitude takes the following form:

$$f(\hat{\mathbf{p}}, \hat{\mathbf{p}}', k) = \sum_{m, m'=-1}^1 4\pi f_{m,m'}(k) Y_1^m(\hat{\mathbf{p}}) Y_1^{m'*}(\hat{\mathbf{p}}'), \quad (6)$$

where  $k\hat{\mathbf{p}}'$  is the incoming momentum and  $k\hat{\mathbf{p}}$  is the outgoing momentum. We can then define  $v_{m,m'}$  and  $k_{m,m'}^{(\text{eff})}$  from the inverse of  $f_{m,m'}(k)$  as a  $3 \times 3$  matrix and its low-energy expansion,

$$(f^{-1})_{m,m'}(k) = -\frac{1}{v_{m,m'} k^2} + \frac{1}{2} k_{m,m'}^{(\text{eff})} - ik \delta_{m,m'} + O(k^2). \quad (7)$$

Note that if  $f_{m,m'}(k) = 0$  for  $m \neq m'$ , as in ordinary cases,  $1/v_{m,m'}$  rather than  $v_{m,m'}$  vanishes.

We are prepared to show the adiabatic sweep theorem for the  $p$ -wave contact tensor. Here, we temporarily remove the assumption that the Feshbach resonance is axisymmetric; that is, we allow  $\epsilon_{m,m'}$  to have an arbitrary form. Then we can solve the two-body problem and calculate the scattering amplitude to determine  $\epsilon_{m,m'}$  and  $g$  in terms of  $1/v_{m,m'}$  and  $k_{m,m'}^{(\text{eff})}$ . We can use those expressions to obtain

$$\frac{\partial \hat{H}_{\text{det}}}{\partial(-1/v_{m,m'})} = \frac{\hbar^2 |g|^2}{32\pi^2 M} \int d^3\mathbf{r} \hat{\phi}_m^\dagger(\mathbf{r}) \hat{\phi}_{m'}(\mathbf{r}). \quad (8)$$

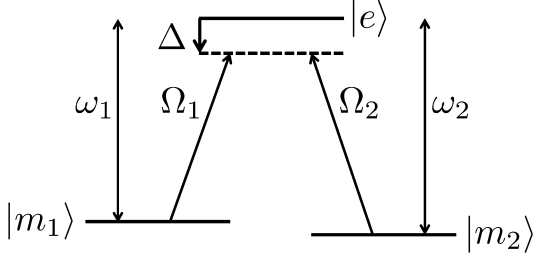


FIG. 1.  $\Lambda$  configuration among  $|m_1\rangle$ ,  $|m_2\rangle$ , and  $|e\rangle$ . Here,  $\omega_i$  ( $i = 1, 2$ ) is the resonance frequency between  $|m_i\rangle$  and  $|e\rangle$ , and  $\Omega_i$  is the complex Rabi frequency which is a complex number. The two closed-channel states are assumed to be at the two-photon resonance detuned by  $\Delta$  from  $|e\rangle$ .

On the other hand, the Hellmann-Feynman theorem reads

$$\frac{\partial E}{\partial(-1/v_{m,m'})} = \left\langle \frac{\partial \hat{H}_{\text{det}}}{\partial(-1/v_{m,m'})} \right\rangle. \quad (9)$$

Combining these with Eq. (5), we obtain the generalized adiabatic sweep theorem (2). After taking the derivative with respect to  $-1/v_{m,m'}$ , we can set the off-diagonal elements of  $\epsilon_{m,m'}$  to be zero as they should be in realistic Feshbach resonance. We emphasize that the off-diagonal components of  $C_{m,m'}$  can be nonzero, in general, even in the absence of the off-diagonal components of  $\epsilon_{m,m'}$ .

No Feshbach resonance can tune  $1/v_{m,m'}$  for  $m \neq m'$ . Here we show that it can be controlled by using a two-photon Raman process. Figure 1 shows the proposed configuration of the molecular levels and lasers. Let  $|e\rangle$  be a diatomic molecular state that couples to the closed-channel states  $|m_1\rangle$  and  $|m_2\rangle$  via lasers of appropriate frequencies and polarizations. We denote the resonant frequencies for  $|m_1\rangle$  and  $|m_2\rangle$  by  $\omega_1$  and  $\omega_2$ , respectively. With the rotating wave approximation, the effective Hamiltonian density  $\mathcal{H}_{\text{Raman}}$  of this three-level system is

$$\hat{\mathcal{H}}_{\text{Raman}} = \begin{pmatrix} \hat{\phi}_{m_1}^\dagger & \hat{\phi}_{m_2}^\dagger & \hat{\phi}_e^\dagger \end{pmatrix} h_{\text{Raman}} \begin{pmatrix} \hat{\phi}_{m_1} \\ \hat{\phi}_{m_2} \\ \hat{\phi}_e \end{pmatrix}, \quad (10)$$

$$h_{\text{Raman}} = \begin{pmatrix} 0 & 0 & \Omega_1^*/2 \\ 0 & 0 & \Omega_2^*/2 \\ \Omega_1/2 & \Omega_2/2 & \Delta \end{pmatrix}, \quad (11)$$

where  $\hat{\phi}_e$  is the bosonic annihilation operator of molecules in  $|e\rangle$ ,  $\Omega_i$  ( $i = 1, 2$ ) is the complex Rabi frequency, and  $\Delta$  is the detuning. For the moment, the spatial argument is omitted from the bosonic annihilation operators. The eigenvalues of  $h_{\text{Raman}}$  is  $0, \frac{1}{2}(\Delta \pm \sqrt{\Delta^2 + |\Omega_1|^2 + |\Omega_2|^2})$ , which we denote by  $\omega_0$  and  $\omega_\pm$  and the corresponding molecular states by  $|\bar{0}\rangle$  and  $|\pm\rangle$ . The eigenvectors of  $h_{\text{Raman}}$  are, apart from the normalization,  $(\Omega_2, -\Omega_1, 0)^T$

and  $(\Omega_1^*, \Omega_2^*, 2\omega_\pm)^T$  for  $\omega_0$  and  $\omega_\pm$ , respectively. We can rewrite  $\mathcal{H}_{\text{Raman}}$  as

$$\hat{\mathcal{H}}_{\text{Raman}} = \omega_0 \hat{\phi}_0^\dagger \hat{\phi}_0 + \omega_- \hat{\phi}_-^\dagger \hat{\phi}_- + \omega_+ \hat{\phi}_+^\dagger \hat{\phi}_+, \quad (12)$$

where  $\hat{\phi}_0$  and  $\hat{\phi}_\pm$  are the bosonic field operators corresponding to  $|\bar{0}\rangle$  and  $|\pm\rangle$ . Among them,  $\hat{\phi}_0$  and  $\hat{\phi}_-$  are adiabatically connected to  $\hat{\phi}_{m_1}$  and  $\hat{\phi}_{m_2}$  in the weak field limit. Now, suppose that no molecules are in  $|e\rangle$  at the initial time and that the intensities of the lasers are adiabatically ramped up. Then we can ignore the third term in Eq. (12), and for weak laser fields such that  $|\Omega_1| \ll \Delta$  and  $|\Omega_2| \ll \Delta$ , we obtain

$$\hat{\mathcal{H}}_{\text{Raman}} \simeq \frac{1}{4\Delta} \left( |\Omega_1|^2 \hat{\phi}_{m_1}^\dagger \hat{\phi}_{m_1} + |\Omega_2|^2 \hat{\phi}_{m_2}^\dagger \hat{\phi}_{m_2} + \Omega_2^* \Omega_1 \hat{\phi}_{m_1}^\dagger \hat{\phi}_{m_2} + \Omega_1^* \Omega_2 \hat{\phi}_{m_2}^\dagger \hat{\phi}_{m_1} \right). \quad (13)$$

This amounts to adiabatically sweeping  $\epsilon_{m_1, m_2} = \epsilon_{m_2, m_1}^*$  as  $\Omega_2^* \Omega_1 / 4\Delta$ , and thus to tuning  $1/v_{m_1, m_2}$  for  $m_1 \neq m_2$ .

The same configuration can be used to measure the off-diagonal components of  $C_{m,m'}$ . This time, the lasers are suddenly turned on and the frequencies are set close to the resonance  $\Delta \ll |\Omega_1|, |\Omega_2|$ . If the lifetime of the excited state  $|e\rangle$  is much shorter than that of  $|m_1\rangle$  and  $|m_2\rangle$ , there remain only the molecules in  $|\bar{0}\rangle$  after some time because  $|\bar{0}\rangle$ , so called the dark state, does not contain  $|e\rangle$ . Therefore, by counting the number of the molecules, we obtain  $\langle \hat{\phi}_0^\dagger \hat{\phi}_0 \rangle = \langle \frac{1}{|\Omega_1|^2 + |\Omega_2|^2} |\Omega_2 \hat{\phi}_1 - \Omega_1 \hat{\phi}_2|^2 \rangle$ , and by repeating the measurement with the different amplitudes and the relative phases of  $\Omega_1$  and  $\Omega_2$ , we can determine  $\langle \hat{\phi}_m^\dagger \hat{\phi}_{m'} \rangle$  for all  $m$  and  $m'$ . Once we know  $\langle \hat{\phi}_m^\dagger \hat{\phi}_{m'} \rangle$ , the  $p$ -wave contact tensor is determined by Eq. (5).

In the remaining part, we discuss possible physical situations in which the off-diagonal components of  $C_{m,m'}$  are significant in systems with  $1/v_{m,m'} = 0$  for  $m \neq m'$ . Specifically, we take two examples, a  $p$ -wave superfluid and a normal Fermi gas in an anisotropic trap.

A  $p$ -wave superfluid provides an example which can exhibit the off-diagonal components of the  $p$ -wave contact tensor due to the spontaneous axisymmetry breaking. If we perform the mean field approximation to the Hamiltonian (3) with the order parameter [14, 15, 19]  $B_m \equiv \frac{1}{\sqrt{V}} \langle \hat{b}_{m0} \rangle$ , we can calculate the momentum distribution as

$$n_{\mathbf{p}} \sim \frac{|g|^2}{4p^2} \left| \sum_{m=-1}^1 Y_1^m(\hat{\mathbf{p}}) B_m \right|^2. \quad (14)$$

From this expression, we can see that the  $p$ -wave contact tensor has the following mean-field contribution:

$$C_{m,m'}^{\text{MF}} = \frac{|g|^2}{4} B_m B_{m'}^*. \quad (15)$$

This takes the diagonal form if and only if the system is in the  $p$  or  $p + ip$  superfluid phase. In the presence of moderate anisotropy of the  $p$ -wave Feshbach resonance, it has

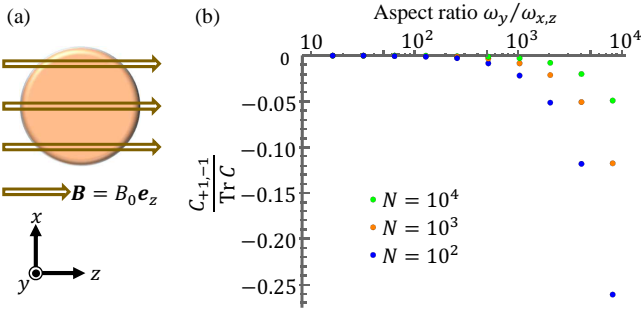


FIG. 2. (Color online) Off-diagonal component of the  $p$ -wave contact tensor in a Fermi gas in a pancake-shaped trap. (a) Configuration of the trap and magnetic field. The gas is tightly confined in the  $y$  direction. (b) Calculated  $C_{+1,-1}$  normalized by  $\text{Tr } C$  with several aspect ratio at  $T = 2T_F$ . We use the interaction parameters of the  $m = \pm 1$  resonance of the  $|F = 9/2, m_F = -7/2\rangle$  state of  $^{40}\text{K}$  at  $B_0 = 198.3\text{ G}$ . The number of particles is  $10^2$  (blue dots),  $10^3$  (orange dots), and  $10^4$  (green dots).

been shown that a Fermi gas described by the Hamiltonian (3) undergoes the spontaneous breaking of the axial symmetry in superfluid phase and a phase transition into the  $p + i\beta p$  superfluid, which is neither the  $p$  nor  $p + ip$  superfluid [14, 15]. Therefore, the off-diagonal components of  $C_{m,m'}$  appear in the  $p$ -wave superfluid with an anisotropic Feshbach resonance, and the emergence of the off-diagonal components signals the transition from the  $p$  to the  $p + i\beta p$  superfluid.

A normal Fermi gas in an anisotropic trap may also reveal the off-diagonal  $p$ -wave contact, at least from the symmetry argument. On the other hand, to the extent that the local-density approximation (LDA) is valid, the off-diagonal components may be zero or very small because the assumption of the local uniformity in the LDA implies that the  $p$ -wave contact tensor density is diagonal at each point in space. Therefore, the off-diagonal  $p$ -wave contact tensor in the normal phase is an indication that the LDA breaks down.

To examine what happens in reality, we calculate the  $p$ -wave contact tensor up to the second order in the cluster expansion [20]. Specifically, we consider a Fermi gas in a pancake-shaped harmonic trap with  $\omega_y > \omega_x = \omega_z = 100\text{ Hz}$  and with an external magnetic field parallel to the  $z$  axis to control the Feshbach resonance, as depicted in Fig. 2(a). The temperature is fixed at  $T = 2T_F$ . We take the interaction parameters for the Feshbach resonance at  $B = 198.3\text{ G}$  in the  $m = \pm 1$  channel in the  $|F = 9/2, m_F = -7/2\rangle$  state of  $^{40}\text{K}$  [13]. We assume that the atoms are non-interacting in the  $m = 0$  channel to avoid a large contribution from the  $m = 0$  dimer; this assumption is consistent with the experimental observation that the  $p$ -wave contact does not grow on the BEC side of the Feshbach resonance on the experimental time scale [12].

Figure 2(b) shows the calculated  $C_{+1,-1} = C_{-1,+1}^*$  normalized by the trace of the  $p$ -wave contact tensor  $\text{Tr } C$  with the varying aspect ratio and the total number of atoms including the atoms in both open and closed channels. The other off-diagonal components are zero due to the symmetry of the configuration. The minus sign implies that the correlation in the tightly confined  $y$  direction is weaker than that in the  $x$  direction. For the small aspect ratio,  $C_{+1,-1}$  is vanishingly small. Indeed, the tighter trap frequency  $\omega_y$  exceeds the temperature  $T$  at the aspect ratio  $\sim 70$  for  $N = 10^2$  and  $\sim 700$  for  $N = 10^4$ ; only for the higher aspect ratio than these does the growth of  $C_{+1,-1}$  become visible. This implies that at this temperature, the quasi-two-dimensional regime should exhibit a significant amount of  $C_{+1,-1}$ . Note that the ratio  $C_{+1,-1}/\text{Tr } C$  depends on  $N$  only through the temperature  $T = 2(\omega_x\omega_y\omega_z/6N)^{1/3}$  in this approximation, and that the smaller  $N$  corresponds to the lower temperature. Thus one may infer that a larger  $C_{+1,-1}/\text{Tr } C$  can be obtained in the current experimental regime of  $T \sim 0.2T_F$ . At that low temperature, however, the current approximation breaks down, and another method is needed.

In conclusion, we have discussed the universal properties of a Fermi gas with a resonant interaction. We have introduced the  $p$ -wave contact tensor, which characterizes both short-range correlations and thermodynamics of such a system. We have generalized the adiabatic sweep theorem applicable to all of the nine components of the  $p$ -wave contact tensor. As the theorem is stated in terms of the parameters characterizing the unconventional type of the  $p$ -wave interaction, we have proposed a Raman process in the  $\Lambda$  scheme to manipulate these parameters and to measure the  $p$ -wave contact tensor. We have also investigated the  $p$ -wave contact tensor in the  $p$ -wave superfluid and a  $p$ -wave Fermi gas in a pancake-shaped trap, where the axial symmetry is broken either spontaneously or externally. In the superfluid phase, the appearance of the off-diagonal component implies the predicted transition from the  $p$  phase to the  $p + i\beta p$  phase. We have argued the possibility that due to the beyond-LDA effect, the off-diagonal components can be observed in an anisotropic trap, even in the normal phase.

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*Note added.* After completion of this work, we became aware of a closely related work by Zhang, He, and Zhou [21], in which they take into account the scattering

in all partial waves. Our conclusions agree where overlapping.

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